MATH 424

HOMEWORK 12

1) Prove that the functions in a and b are harmonic in $\mathbb{C}$

a) $\phi(x,y) = \sin x \sinh y$

b) $\phi(x,y) = e^{x^2-y^2} \sin(2xy)$

2) Find the conjugates of the harmonic functions given in a and b.

a) $u = x^2 - (y-1)^2$

b) $u = \frac{1}{2}\log(x^2+y^2)$

3) Show that Poisson's integral formula for the upper half plane y > 0 is:

$$u(z) = \frac{y}{\pi}\int_{-\infty}^{\infty} \frac{u(t)dt}{(t-x)^2+y^2}, \qquad z = x+iy.$$

Hint: Use the Conformal Map $g(z) = \frac{z-z_0}{z-\bar{z}_0}$, with $g(z_0) = 0$

4) Show that Poisson's integral formula for the first quadrant is:

$$u(z) = \frac{4xy}{\pi}\left[\int_0^{\infty} \frac{tu(t)dt}{t^4-2t^2(x^2-y^2)+(x^2+y^2)^2} + \int_0^{\infty} \frac{tu(it)dt}{t^4+2t^2(x^2-y^2)+(x^2+y^2)^2}\right]$$

Hint: use the conformal map $g(z) = \frac{(z^2-z_0^2)}{(z^2-\bar{z}_0^2)}$, with $g(z_0) = 0$.

5. Find the inverses of the Laplace transforms given in the following questions, a, b, c, and d.

Assume that each transform is defined in the half plane Re $s > a$ and b is real.

a) $\frac{1}{(s^2+a^2)^2}$

b) $\frac{1}{(s+a)^4}$

c) $\frac{e^{-bs}}{(s+a)^3}$

d) $\frac{e^{-bs}}{s(s^2+a^2)}$