

MATH 424
HOMEWORK 8
SPRING 2007

1. Evaluate the integrals:

$$a) \int_{-\infty}^{\infty} \frac{\sin \pi x}{x^5 - x} dx = \frac{\pi}{2} (e^{-\pi} - 3)$$

$$b) \int_{-\infty}^{\infty} \frac{\sin x}{x} \frac{x^2 + a^2}{x^2 + b^2} dx = \frac{\pi}{b^2} [a^2 + e^{-b} (b^2 - a^2)], \quad a, b > 0$$

$$c) \int_0^{\infty} \frac{\sin ax}{x(x^2 + b^2)^2} dx = \frac{\pi}{2b^4} \left[1 - \frac{e^{-ab}}{2} (ab + 2) \right], \quad a, b > 0$$

2. Prove the identity:

$$PV \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{e^{itx}}{x} dx = \begin{cases} \frac{1}{2}, & t > 0, \\ 0, & t = 0, \\ -\frac{1}{2}, & t < 0, \end{cases}$$

If we add $\frac{1}{2}$ to this function, we obtain the “impulse function”, often found in engineering books, representing a sudden switch-in of current into an open circuited electric line.

3. Evaluate the integral:

$$a) \int_{-\infty}^{\infty} \frac{e^{ay} dy}{1 + be^{-y}} = \frac{-\pi b^a}{\sin \pi a}, \quad 0 > a > -1, b > 0, \text{ Hint: set } x = e^y$$

$$b) \int_0^{\infty} \frac{x^a dx}{x^2 + 2x \cos \theta + 1} = \frac{\pi}{\sin \pi a} \frac{\sin \theta a}{\sin \theta}, \quad 1 > a > -1, \pi > \theta > -\pi$$

$$c) \int_0^{\infty} \frac{x^a dx}{x^3 + b^3} = \frac{2\pi b^{a-2}}{3 \sin \pi a} \left[\cos \frac{\pi}{3} (1 - 2a) - \frac{1}{2} \right], \quad 2 > a > -1, b > 0$$

4. Evaluate the integral

$$\int_0^{\infty} \frac{x \cos ax}{\sinh x} dx = \frac{\pi^2}{4} \operatorname{sech}^2 \frac{a\pi}{2}, \quad a \text{ real}$$