

Math 424
Homework 87

1. Find the residue at all of the singularities in the complex plane for the following functions:

a) $f(z) = z \exp(\frac{1}{z})$

b) $f(z) = (z - 1) \exp(\frac{1}{z})$

2. Evaluate the integrals

a) $\int_{|z-1|=2} \frac{dz}{z^n(z^2+1)}$

b) $\int_{|z-\frac{1}{2}|=1} \frac{\sin(z)}{(z^3+z)} dz$

3. Evaluate the definite integrals

a) $\int_0^{2\pi} \frac{d\theta}{a^2 \cos^2 \theta + b^2 \sin^2 \theta} \quad a, b > 0$

b) $\int_0^{2\pi} \frac{d\theta}{1 - 2a \cos \theta + a^2}$

$|a| < 1$ and $|a| > 1$

4. Evaluate the following indefinite integrals

a) $\int_0^{\infty} \frac{x^2 dx}{(x^2+a^2)^2} \quad a > 0$

b) $\int_{-\infty}^{\infty} \frac{x^3 \sin(ax) dx}{(x^2+b^2)^2}$

$a, b > 0$