# MATH 424

## HOMEWORK 6

1. Obtain the Maclaurin series given in:

   a) $\cos z = \sum_{n=0}^{\infty} (-1)^n \frac{z^{2n}}{(2n)!}, \qquad |z| < \infty$

   b) $\cosh z = \sum_{n=0}^{\infty} \frac{z^{2n}}{(2n)!}, \qquad |z| < \infty$

2. Expand the given functions in a Taylor series about $z_0$. Indicate the largest disk where the representation is valid:

   a) $f(z) = \frac{1}{1-z}$, $\qquad z_0 = -1$

   b) $f(z) = \cos z$, $\qquad z_0 = \frac{\pi}{2}$

   c) $f(z) = \log z$, $\qquad z_0 = -1$

3. Find the order of the zero at $z = 0$ for the functions given:

   a) $z^2(\cos z - 1)$ $\qquad$ b) $z - \tan z$

4. Represent the function $(z^3 - z)^{-1}$ as a Laurent series in the regions given:

   a) $0 < |z| < 1$ $\qquad$ b) $0 < |z - 1| < 1$

5. Find the Laurent series of the function $z/(z^2 + z - 2)$ in the regions given:

   a) $|z| < 1$ $\qquad$ b) $0 < |z + 2| < 3$ $\qquad$ c) $|z| > 2$

6. Find the Laurent series of the functions in the region $0 < |z - 1| < 1$ given in:

   a) $\frac{1}{z-1} \sin \frac{1}{z}$ $\qquad$ b) $\sin \frac{1}{z(z-1)}$