MATH 424
HOMEWORK 4

1. Evaluate the following definite integrals i) $\int \bar{z}dz$ and ii) $\int ydz$
   a) along the directed line segment from 0 to $1-i$
   b) around the circle $|z|=1$
   c) around the circle $|z-a|=R$

2. Evaluate the integral $\int (z-a)^n dz$, n an integer, around the circle $|z-a|=R$. (Note: the case $n=-1$ is special)

3. a) Show that $\int_{\partial G} x\, dz = iA$
   b) Show that $\int_{\partial G} \bar{z}\, dz = 2iA$

4. Obtain the integrals:

$$\int_0^T \cos at \cosh bt dt = \frac{a \sin aT \cosh bT + b \cos aT \sinh bT}{a^2+b^2}$$

$$\int_0^T \sin at \sinh bt dt = \frac{b \sin aT \cosh bT - a \cos aT \sinh bT}{a^2+b^2}$$

by integrating $f(z)=\cos z$ along the line segment from 0 to $(a+ib)T$.

5. Prove that:

$$\int_{-\infty}^{\infty} e^{-kx^2} \cos ax\, dx = \sqrt{\frac{\pi}{k}} e^{-a^2/4k}, \qquad k>0, \ a \text{ real},$$

by using the same procedure as in Example 3, discussed in class, with the function $f(z)=e^{-kz^2}$. Check your answer by changing variables.

6. Prove Dirichlet's integral:

$$\int_0^{\infty} \frac{\sin x}{x} dx = \frac{\pi}{2},$$

by integrating $f(z)=e^{iz}/z$ along the boundary of the set $r \le |z| \le R$, $0 \le \arg z \le \pi$. Check your answer by changing variables and comparing with Example 4, discussed in class.

7. Let $\gamma : z(t) = 2e^{it}+1, 0 \le t \le 2\pi$. Evaluate the integrals in a) and b).
   a) $\int_\gamma \frac{e^z}{z} dz$
   b) $\int_\gamma \frac{\sin z}{z^2+1} dz$

   Let $\gamma : z(t) = 2e^{it}+1, 0 \le t \le 2\pi$. Evaluate the integrals in c) and d).
   c) $\int_\gamma \frac{e^z}{z^2} dz$
   d) $\int_\gamma \frac{\sin z}{(z^2+1)^2} dz$