Math 424/524

Homework 3

1) Show that $f(z) = \bar{z}$ and $f(z) = |z|$ are continuous for all z.

2) Show that $f: A \subset \mathbb{C} \to \mathbb{C}$ is continuous if and only if $z_n \to z_0 \in A \Rightarrow f(z_n) \to f(z_0)$

3) Determine where the following functions are analytic and compute their derivatives.

i) $(z+1)^3$

ii) $z + \frac{1}{z}$

iii) $\frac{3z-1}{3-z}$

iv) $\frac{1}{(z+\frac{1}{z})^2}$

4) Verify the Cauchy-Riemann equations for

$$f(z) = z^2 + 3z + 2$$

5) Prove that $f(z) = |z|$ is not analytic

6) Show, by changing variables, that the Cauchy-Riemann equations in polar coordinates become

$$\partial_r u = \frac{1}{r} \partial_\theta v$$

$$\partial_r v = -\frac{1}{r} \partial_\theta u$$

7) Define the symbol $\frac{\partial f}{\partial \bar{z}}$ by

$$\frac{\partial f}{\partial \bar{z}} = \frac{1}{2} \left(\frac{\partial f}{\partial x} - \frac{1}{i} \frac{\partial f}{\partial y} \right)$$

Show that the Cauchy-Riemann equations are equivalent to

$$\frac{\partial f}{\partial \bar{z}} = 0.$$