Math 424/524

Homework 2

1. Solve
   a) $\sin z = \frac{3}{4} + \frac{i}{4}$
   b) $\sin z = 4$

2. Find all the values of:
   a) $(-i)^i$ c) $\log(1+i)$
   b) $(1+i)^{1+i}$ d) $\log(-i)$

3. Prove the identity

$$z = \tan\left[\frac{1}{i}\log\left(\frac{1+iz}{1-iz}\right)^{\frac{1}{2}}\right]$$

4. Use the equation $\sin z = \sin x \cosh y + i \sinh y \cos x$ where z=x+iy to prove that $|\sinh y| \leq |\sin z| \leq |\cosh y|$

5. Using polar coordinates, show that $z \mapsto z + 1/z$ maps the circle |z|=1 to the interval [-2,2] on the x-axis.

6. For each of the following sets, state (i) whether or not it is open and (ii) whether or not it is closed.

a) $\{z \mid \operatorname{Im} z > 2\}$ b) $\{z \mid 1 \leq |z| \leq 2\}$ c) $\{z \mid -1 < \operatorname{Re} z \leq 2\}$