

[ Solution Set  $P_{en}$  11-26-90 ]

LT

⑮  $\nabla^2 T = 0$      $T(0, y) = T(l, y) = 0$  ;  $T(x, 0) = g(x)$  ,  $T(x, d) = P(x)$

$$T = X(x) Y(y) \Rightarrow X'' Y + Y'' X = 0$$

$$\frac{X''}{X} = -\frac{Y''}{Y} = -k^2 = -\left(\frac{n\pi}{l}\right)^2$$

$$* \Rightarrow X_n = A_n \sin\left(\frac{n\pi x}{l}\right), \quad Y_n = B_n e^{\frac{n\pi y}{l}} + C_n e^{-\frac{n\pi y}{l}}$$

$$\text{or } = A_n \sinh\left[\frac{n\pi y}{l}\right] + C_n \sinh\left[\frac{n\pi(y-d)}{l}\right]$$

$$\Rightarrow T_n = \left[ a_n \sinh\left(\frac{n\pi y}{l}\right) + b_n \sinh\left(\frac{n\pi(d-y)}{l}\right) \right] \sin\left(\frac{n\pi x}{l}\right)$$

Finding constants:

$y=0$      $\sum b_n \sinh\left(\frac{n\pi d}{l}\right) \sin\left(\frac{n\pi x}{l}\right) = g(x)$

$$\Rightarrow b_n = \frac{2}{l \sinh\left(\frac{n\pi d}{l}\right)} \int_0^l g(x) \sin\left(\frac{n\pi x}{l}\right) dx$$

$y=d$      $\sum a_n \sinh\left(\frac{n\pi d}{l}\right) \sin\left(\frac{n\pi x}{l}\right) = P(x)$

$$\Rightarrow a_n = \frac{2}{l \sinh\left(\frac{n\pi d}{l}\right)} \int_0^l P(x) \sin\left(\frac{n\pi x}{l}\right) dx$$

⑯ as above, except  $y \rightarrow \infty \Rightarrow Y_n(y) \rightarrow 0$

$$\Rightarrow B_n \text{ in } * \text{ above is } 0 \quad (A_n = 0)$$

$$\Rightarrow T = \sum_{n=1}^{\infty} c_n e^{-\frac{n\pi y}{l}} \sin\left(\frac{n\pi x}{l}\right)$$

$$c_n = \frac{2}{l} \int_0^l F(x) \sin\left(\frac{n\pi x}{l}\right) dx$$

From (14)

$$\begin{aligned}
 (16) \quad T(x, y) &= \sum_{n=1}^{\infty} a_n \sinh\left(\frac{n\pi(d-y)}{L}\right) \sin\left(\frac{n\pi x}{L}\right) \\
 &= \sum_{n=1}^{\infty} c_n \frac{\sinh\left(\frac{n\pi(d-y)}{L}\right)}{\sinh\left(\frac{n\pi d}{L}\right)} \sin\left(\frac{n\pi x}{L}\right) \\
 &= \sum_{n=1}^{\infty} c_n \frac{e^{\frac{n\pi}{L}(d-y)} - e^{-\frac{n\pi}{L}(d-y)}}{e^{\frac{n\pi}{L}d} - e^{-\frac{n\pi}{L}d}} \sin\left(\frac{n\pi x}{L}\right) \\
 &= \sum_{n=1}^{\infty} c_n \frac{e^{-\frac{n\pi}{L}y} - e^{\frac{n\pi}{L}y} e^{-2\frac{n\pi}{L}d}}{1 - e^{-2\frac{n\pi}{L}d}} \sin\left(\frac{n\pi x}{L}\right) \left\{ \begin{array}{l} \rightarrow 0 \text{ as } d \rightarrow \infty \\ \rightarrow 0 \text{ as } d \rightarrow 0 \end{array} \right. \\
 \Rightarrow T &= \sum_{n=1}^{\infty} c_n e^{-\frac{n\pi}{L}y} \sin\left(\frac{n\pi x}{L}\right)
 \end{aligned}$$

17<sup>+18</sup> will be posted soon!