9.18

a. From 17. b,
$$c_n = \frac{2}{n\pi kh}\int_0^{l} g(x)\sin\frac{n\pi x}{l}\,dx$$
$$= \frac{2}{n\pi kh}\int_{x_0-\epsilon}^{x_0+\epsilon} \frac{\sin\frac{n\pi x}{l}}{2\epsilon}\,dx$$
$$= \frac{-l}{n^2\pi^2 kh\epsilon}\left[\cos\left(\frac{n\pi}{l}(x_0+\epsilon)\right) - \cos\left(\frac{n\pi}{l}(x_0-\epsilon)\right)\right]$$
$$= \frac{-l}{n^2\pi^2 kh\epsilon}\Big[\cos\left(\frac{n\pi}{l}x_0\right)\cos\left(\frac{n\pi}{l}\epsilon\right) - \sin\left(\frac{n\pi}{l}x_0\right)\sin\left(\frac{n\pi}{l}\epsilon\right)$$
$$- \cos\left(\frac{n\pi}{l}x_0\right)\cos\left(\frac{n\pi}{l}\epsilon\right) - \sin\left(\frac{n\pi}{l}x_0\right)\sin\left(\frac{n\pi}{l}\epsilon\right)\Big]$$
$$= \underline{\frac{2l}{n^2\pi^2 kh\epsilon}\sin\left(\frac{n\pi}{l}x_0\right)\sin\left(\frac{n\pi}{l}\epsilon\right)}$$

b. For small ϵ,
$$c_n \approx \frac{2l}{n^2\pi^2 kh\epsilon}\sin\left(\frac{n\pi}{L}x_0\right)\frac{n\pi\epsilon}{L}$$
$$= \frac{2}{n\pi kh}\sin\frac{n\pi x_0}{L}$$

From 17. b,
$$T(x,y) = \sum_{n=1}^{\infty} c_n e^{-n\pi y/L}\sin\left(\frac{n\pi x}{L}\right)$$
$$= \underline{\frac{2}{\pi kh}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi x_0}{L}\right)\sin\left(\frac{n\pi x}{L}\right)e^{-n\pi y/L}}$$

c. Let the above $\equiv V(x,y;x_0)$. Expanding the solution to 17. b,
$$T(x,y) = \int_0^{l} dx'\,\frac{2g(x')}{\pi kh}\sum_{n=1}^{\infty}\frac{1}{n}\sin\left(\frac{n\pi x'}{L}\right)e^{-n\pi y/L}\sin\left(\frac{n\pi x}{L}\right)$$

Replacing the dummy variable $x' \to x_0$ clearly yields
$$T(x,y) = \int_0^{l} dx_0\, V(x,y;x_0)\, G(x_0)$$