Surface Integral

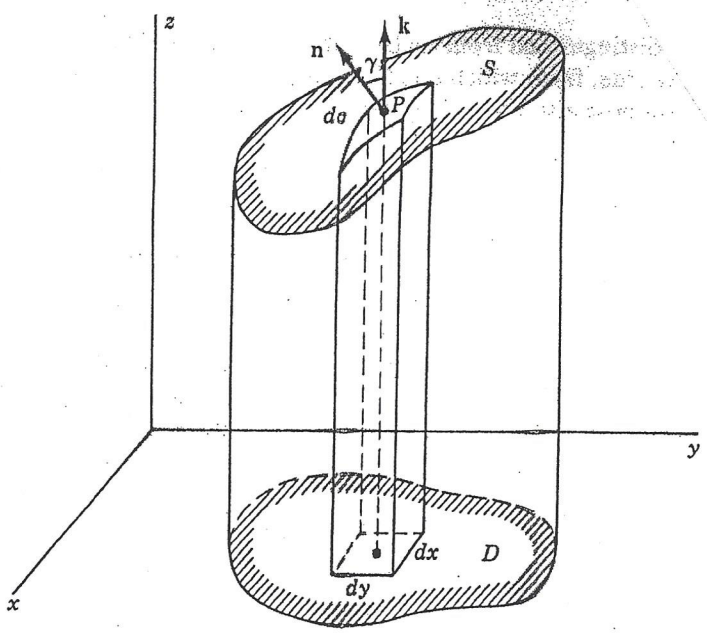

Figure 6.11

Unit vector normal to surface element $d\sigma$: $\hat{n} = \pm \dfrac{\vec{\nabla}\varphi}{|\vec{\nabla}\varphi|}$

" " " " x-y plane $\hat{k}$

Surface vector element $\vec{d\sigma} = \hat{n}\, d\sigma$

Projection of $\vec{d\sigma}$ onto x-y-plane : $|\hat{n}\cdot\hat{k}|\, d\sigma = dx\,dy$

But $\hat{n}\cdot\hat{k} = \cos\gamma$, because $|\hat{n}| = |\hat{k}| = 1$

↳ direction cosine

$\hat{k}$ component of $\hat{n}$

$$\therefore \quad \hat{n}\cdot\hat{k} = \cos\gamma = \pm \frac{\partial\varphi/\partial z}{\sqrt{\left(\frac{\partial\varphi}{\partial x}\right)^2 + \left(\frac{\partial\varphi}{\partial y}\right)^2 + \left(\frac{\partial\varphi}{\partial z}\right)^2}}$$