and $\frac{s}{s-s_0} M$ is bounded as $s \to \infty$.

SF(s) not bounded as $s \to \infty$

Thus functions such as 1, $\frac{s}{s+1}$, $\frac{1}{\sqrt{s}}$ + $\sin s$ cannot be transforms of functions $f(t)$ satisfying the conditions stated.

→ Insert A

Calculating Inverse Laplace Transforms

A Partial Fraction Expansions

We want to write quotients of polynomials in s as a sum of simpler quotients.

e.g

$$\frac{1}{s^2(s^2-4)} + \frac{(s-2)}{(s^2+4)} = -\frac{1}{4}\cdot\frac{1}{s^2} + \frac{15}{16}\cdot\frac{1}{s+2} + \frac{1}{16}\cdot\frac{1}{s-2}$$

All terms on the right have easily recognizable inverses.

Suppose that we have a quotient of polynomials,

$$\frac{F(s)}{G(s)} = \frac{a_0 + a_1 s + \cdots + a_p s^p}{b_0 + b_1 s + \cdots b_r s^r}$$

such that:

1. All coefficients are real.
2. All common factors have been divided out [so that F(s) and G(s) have no common root]
3. G(s) is of higher degree than F(s) [otherwise divide G(s) into F(s)] i.e $p < r$.

Now $G(s)$ can always be split into a product of factors, each of the form

$$(s-a)^m$$

(linear factors) OR

$$(s^2+ps+q)^n$$

(quadratic factors).

We can write $F(s)/G(s)$ as a sum of simpler fractions by:

1. Assigning to each $(s-a)^m$ factor of $G(s)$ a sum of terms
$$\frac{A_1}{(s-a)} + \frac{A_2}{(s-a)^2} + \cdots + \frac{A_m}{(s-a)^m} \;;$$

2. Assigning to each $(s^2+ps+q^2)^n$ factor of $G(s)$ a sum of terms
$$\frac{B_1 s + C_1}{(s^2+ps+q)} + \frac{B_2 s + C_2}{(s^2+ps+q)^2} + \cdots + \frac{B_n s + C_n}{(s^2+ps+q)^n} \;;$$

3. Solving for the constants $A_1, \ldots, B_1, \ldots, C_1, \ldots$

Examples:

Find $\mathcal{L}^{-1}\left[\dfrac{s+4}{s^2-s-6}\right]$

write
$$\frac{s+4}{(s-3)(s+2)} = \frac{A}{s-3} + \frac{B}{s+2}$$

$$\frac{s+4}{s^2-s-6} = \frac{(A+B)s + 2A - 3B}{s^2-s-6}$$

Equality can hold only is like power of s ~~are equal~~ on both sides are equal

$\Rightarrow$ s^1: $\quad A+B = 1$

s^0: $\quad 2A-3B = 4$

Solve $\quad A = 7/5 \;,\; B = -2/5$

So $$\frac{s+4}{s^2-s-6} = \frac{7/5}{s-3} - \frac{2/5}{s+2}$$

$$\therefore \mathcal{L}^{-1}\left[\frac{s+4}{s^2-s-6}\right] = \frac{7}{5}e^{3t} - \frac{2}{5}e^{-2t}$$

Exercise: Find $\mathcal{L}^{-1}\left[\dfrac{s^2+2s-4}{s^3-5s^2+2s+8}\right]$

$\mathcal{L}^{-1}\left[\dfrac{2s+1}{(s^2+4)(s-3)}\right]$, $\mathcal{L}^{-1}\left[\dfrac{2s^2-s}{(s^2+4)^2}\right]$.