Differentiation Formulas

The following identities are of frequent use:

$$\boldsymbol{\nabla} \cdot \boldsymbol{\varphi}\mathbf{u} = \boldsymbol{\varphi}\boldsymbol{\nabla} \cdot \mathbf{u} + \mathbf{u} \cdot \boldsymbol{\nabla}\boldsymbol{\varphi} ,$$

$$\boldsymbol{\nabla} \times \boldsymbol{\varphi}\mathbf{u} = \boldsymbol{\varphi}\boldsymbol{\nabla} \times \mathbf{u} + \boldsymbol{\nabla}\boldsymbol{\varphi} \times \mathbf{u} ,$$

$$\boldsymbol{\nabla} \cdot \mathbf{u} \times \mathbf{v} = \mathbf{v} \cdot \boldsymbol{\nabla} \times \mathbf{u} - \mathbf{u} \cdot \boldsymbol{\nabla} \times \mathbf{v} ,$$

$$\boldsymbol{\nabla} \times (\mathbf{u} \times \mathbf{v}) = \mathbf{v} \cdot \boldsymbol{\nabla}\mathbf{u} - \mathbf{u} \cdot \boldsymbol{\nabla}\mathbf{v} + \mathbf{u}(\boldsymbol{\nabla} \cdot \mathbf{v}) - \mathbf{v}(\boldsymbol{\nabla} \cdot \mathbf{u}) ,$$

$$\boldsymbol{\nabla}(\mathbf{u} \cdot \mathbf{v}) = \mathbf{u} \cdot \boldsymbol{\nabla}\mathbf{v} + \mathbf{v} \cdot \boldsymbol{\nabla}\mathbf{u} + \mathbf{u} \times (\boldsymbol{\nabla} \times \mathbf{v}) + \mathbf{v} \times (\boldsymbol{\nabla} \times \mathbf{u}) ,$$

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla}\boldsymbol{\varphi}) = \text{curl grad } \boldsymbol{\varphi} = \mathbf{0} ,$$

$$\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla} \times \boldsymbol{u}) = \text{div curl } \mathbf{u} = 0 ,$$

$$\boldsymbol{\nabla} \times (\boldsymbol{\nabla} \times \mathbf{u}) = \text{curl curl } \mathbf{u} = \boldsymbol{\nabla}(\boldsymbol{\nabla} \cdot \mathbf{u}) - \boldsymbol{\nabla} \cdot \boldsymbol{\nabla}\mathbf{u} \boldsymbol{,}$$

$$= \text{grad div } \mathbf{u} - \boldsymbol{\nabla}^{\mathbf{2}}\mathbf{u}$$

$$\boldsymbol{\nabla} \cdot (\boldsymbol{\nabla}\boldsymbol{\varphi}_{\mathbf{1}} \times \boldsymbol{\nabla}\boldsymbol{\varphi}_{\mathbf{2}}) = 0$$